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**POWER FLOW FROM A PLASMA-HAVING COMPLEX  
ELECTROACOUSTIC WAVE IMPEDANCE**

**Robert L. Galloway**  
National Bureau of Standards

**TECHNICAL REPORT NO. RADC-TR-44-33**  
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**Robert L. Gellera**

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## FOREWORD

This work comprises a special report on U.S. Air Force, Rome Air Development Center, Research and Technology Division, Griffiss Air Force Base, New York, Contract No. AF30(602)-3836, Project 5582, Task 558201, I.T.S.A. Project No. 5101421. The computations reported in this report were performed in part on an IBM-7090 computer and in part on a CDC-3600 computer.


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# POWER FLOW FROM A PLASMA HAVING COMPLEX ELECTROACOUSTIC WAVE IMPEDANCE

Robert L. Gallawa

The complex electroacoustic wave impedance is examined and is related to power coupling at a plasma-vacuum boundary via the plane wave solution of Maxwell's equations. The reactive component of impedance is found to be important at frequencies less than the plasma frequency, where it contributes a significant amount to power flow. Thus at those frequencies serious errors can be introduced by assuming that the electroacoustic power is made up of the powers in the incident and reflected waves only.

**Key words:** Electroacoustic, electromagnetic, impedance, plasma, power coupling, wave propagation.

## 1. Introduction

The problem of understanding the coupling mechanism which governs the conversion of electromagnetic energy at a plasma density discontinuity has received considerable attention in recent years. Work has been done on the problem of reflection from and transmission into a plasma half-space [Wait, 1964a, b; Hessel, et al., 1962; Kritz and Mintzer, 1960], as well as the reciprocal problem that is encountered when the wave is incident from the plasma onto the plasma-vacuum boundary [Field, 1956; Jöhler, 1964; Gallawa, 1965]. In the latter problem one may consider incident longitudinal (or transverse) waves onto the boundary, there being coupling to the transverse (or longitudinal) wave by the boundary. In particular an incident longitudinal wave generates reflected and transmitted transverse waves, as well as a reflected longitudinal wave. The

problem is complicated by coupling between the two types of waves if a magneto-static field permeates the region.

The mechanism by which a compression (longitudinal) wave establishes an electromagnetic wave at a boundary is now fairly well understood. The problem of power conversion introduces an insight which is not readily available when work is restricted to the wave magnitudes. The purpose of this paper is to study the coupling at a plasma-vacuum boundary by introducing the complex intrinsic wave impedance. The reactive component thereof will be shown to contribute to power transmitted by virtue of an interaction of incident and reflected waves. The problem is believed to be particularly important in view of the recent work by Chen [1965] in which he showed that a space vehicle illuminated by an electromagnetic wave can excite an electroacoustic wave which may enhance the radar return considerably.

## 2. Equations for the Unbounded Homogeneous Plasma

To introduce important concepts and equations consideration is given first to the case of an unbounded homogeneous plasma. The medium will be taken to be a one-component, uniform electron fluid; i. e. heavy ionic motion will be neglected. Collisions between electrons and neutral particles will be accounted for by introducing a constant collision frequency,  $\nu$ . In addition finite compressibility will be considered. The wave magnitudes to be introduced will be assumed to be very small,



permitting the use of linearized hydrodynamic equations [Oster, 1960],

which are given by

$$mn_0 \left( \mathbf{v} + \frac{\partial}{\partial t} \bar{\mathbf{V}} \right) = n_0 e \bar{\mathbf{E}} - \nabla p, \quad (1)$$

$$u^2 mn_0 \nabla \cdot \bar{\mathbf{V}} = - \frac{\partial p}{\partial t}, \quad (2)$$

$$\nabla \times \bar{\mathbf{E}} = - \mu_0 \frac{\partial \bar{\mathbf{H}}}{\partial t}, \quad (3)$$

$$\nabla \times \bar{\mathbf{H}} = \epsilon_0 \frac{\partial \bar{\mathbf{E}}}{\partial t} + n_0 e \bar{\mathbf{V}}, \quad (4)$$

where  $e$  and  $m$  are the charge and mass of the electron, respectively;

$\mu_0$  and  $\epsilon_0$  are the permeability and permittivity of free space, respectively;

$u$  is the speed of sound in the electron gas,  $u^2 \cong \gamma \frac{KT}{m}$ ,

$K$  = Boltzmann's constant,  $T$  = absolute temperature;  $p$  is the deviation

of the electron pressure from the mean, and  $\bar{\mathbf{V}}$  is the mean electron

velocity;  $n_0$  is the constant electron number density;  $\gamma$  is the ratio of

specific heats;  $\bar{\mathbf{E}}$  and  $\bar{\mathbf{H}}$  are, as usual, the electric and magnetic fields.

For now the steady magnetic induction is assumed to be zero; that additional

generality can be introduced by replacing  $\bar{\mathbf{E}}$  by  $\bar{\mathbf{E}} + \bar{\mathbf{V}} \times \bar{\mathbf{B}}$  in (1).

Without loss of generality a time factor  $\exp(i\omega t)$  will subsequently be

assumed and suppressed. The value of  $\frac{u}{c}$ , where  $c$  is the velocity of

light, is taken to be  $10^{-3}$  throughout this paper. This corresponds to a

temperature of about  $10^4$  degrees Kelvin.

It is convenient to study the propagation parameters by splitting  $\bar{E}$  into longitudinal and transverse components [Hessel and Shmoy, 1961; Hessel, Marcuvitz, and Shmoy, 1962; Chen, 1964; Field, 1956].

Thus let

$$\bar{E} = \bar{E}_l + \bar{E}_t, \quad (5)$$

where

$$\nabla \times \bar{E}_l = 0, \quad (6)$$

$$\nabla \cdot \bar{E}_t = 0. \quad (7)$$

Then by (2), (4), and (7),

$$\nabla \cdot \bar{E}_l = \frac{\rho}{u^2 \epsilon_0 m} p. \quad (8)$$

In addition,

$$\nabla \times \bar{E}_t = -i\omega \mu_0 \bar{H}. \quad (9)$$

From these equations, it is evident that  $\bar{E}_l$  is in the direction of propagation (and hence is longitudinal) while  $\bar{E}_t$  is entirely transverse to the direction of propagation. Clearly there is no magnetic field associated with the longitudinal wave.

There is no difficulty to show [Callawa, 1964] the pertinent equations satisfied by  $\bar{E}_t$  and  $p$ , as

$$\nabla^2 \bar{E}_r + \frac{\omega^2}{c^2} \left[ 1 - \frac{1 - \frac{\omega_p^2}{\omega^2}}{\omega(\nu + i\omega)} \right] \bar{E}_r = 0 \quad (10)$$

$$\nabla^2 p + \frac{\omega^2}{u^2} \left[ 1 - \frac{\omega_p^2}{\omega^2} - i \frac{\nu}{\omega} \right] p = 0 \quad (11)$$

where the (angular) plasma frequency is given by

$$\omega_p = \left[ \frac{n_0 e^2}{m \epsilon_0} \right]^{\frac{1}{2}}$$

The usual Cartesian coordinate interpretation of the  $\nabla^2$  operator is applicable in (10). The pressure wave has an associated longitudinal electric field by virtue of (8).

The waves defined by (10) and (11) have propagation constants  $k_r$  and  $k_l$  given respectively by

$$k_r = \frac{\omega}{c} \left[ 1 - i \frac{1 - \frac{\omega_p^2}{\omega^2}}{\omega(\nu + i\omega)} \right]^{\frac{1}{2}} = \kappa \quad (12)$$

$$k_l = \frac{\omega}{u} \left[ 1 - \frac{\omega_p^2}{\omega^2} - i \frac{\nu}{\omega} \right]^{\frac{1}{2}} = \kappa \frac{c}{u} \left( 1 - i \frac{\nu}{\omega} \right)^{\frac{1}{2}} = \kappa \frac{\nu}{u} \quad (13)$$

where

$$\nu = \left[ \frac{\nu + i\omega}{1 - \frac{\omega_p^2}{\omega^2}} \right]^{\frac{1}{2}} \quad (14)$$

The square roots are to be chosen so that  $\text{Im } k_r < 0$  and  $\text{Im } k_l < 0$ .

### 3. The Electroacoustic Complex Wave Impedance

The complex wave impedance for electromagnetic waves has been used recently [Barlow, 1963] to illustrate power transmission under

standing-wave conditions. Concentration here is on the extension of that concept to include longitudinal waves. To define the necessary impedance break  $\bar{V}$  into components:

$$\bar{V} = \bar{V}_l + \bar{V}_t \quad .$$

where  $\bar{V}_l$  and  $\bar{V}_t$  are associated with the longitudinal and transverse waves respectively. In fact, it is convenient to let

$$m n_0 (\nu + i\omega) \bar{V}_l = n_0 e \bar{E}_l - \nabla p, \quad (15)$$

$$m n_0 (\nu + i\omega) \bar{V}_t = n_0 e \bar{E}_t, \quad (16)$$

If  $p$ ,  $\bar{V}_l$  and  $\bar{E}_l$  are taken to be plane waves, varying as  $\exp(-ik_l x)$ , the intrinsic impedance of the electroacoustic (longitudinal) wave can be defined as

$$\hat{Z} = \frac{V_l}{p} \quad . \quad (17)$$

where  $V_l$  is the  $x$  directed component of  $\bar{V}_l$ . In the case of a pure longitudinal wave, there will be no other components of  $\bar{V}$ . In particular, we may take

$$\bar{E}_l = \bar{a}_l E_{0l} \exp(-ik_l x) \quad , \quad (18)$$

$$p = -ik_l \frac{u^2 m n_0}{e} E_{0l} \exp(-ik_l x) \quad , \quad (19)$$

$$\bar{V}_1 = \bar{a}_1 \frac{(-i\omega\epsilon_1)}{n_1 e} E_{01} \exp(-ik_1 x) . \quad (20)$$

Then,

$$\hat{Z} = \frac{\hat{Z}_0}{\sqrt{1 - \left(\frac{\hat{Z}_0}{u}\right)^2 - i \frac{v}{u}}} . \quad (21)$$

where

$$\hat{Z}_0 = \frac{1}{\rho_1 u} ,$$

$$\rho_1 = n_1 m .$$

In the case of a collisionless nonionized gas, the impedance reduces to that of the usual acoustic wave [Morse, 1949; Moore, 1960]. In the case of a pure transverse wave, the corresponding expression for impedance is [Barlow, 1963]

$$Z = \frac{Z_0}{\sqrt{1 - h - ih \frac{v}{u}}} , \quad (22)$$

where

$$Z = \sqrt{\frac{\mu_1}{\epsilon_1}} ,$$

$$h = \frac{u_1^2}{u^2 + v^2} .$$

In this case, the impedance is associated with the electromagnetic wave. Barlow has treated the ramifications of this impedance extensively, and

so attention will be restricted here to the treatment of longitudinal wave propagation.

In the case of a pressure wave impinging on a density discontinuity, the total wave will be made up of an incident and a reflected component; that is

$$p = p^+ + p^- .$$

where the superscripts + and - refer to the incident and reflected components, respectively. Associated with these waves are the corresponding  $V_t^+$ ,  $V_t^-$ . Thus, for example,

$$p^+ = - ik_t \frac{u^2 m c_s}{\rho} E_{01} \exp(- ik_t x) .$$

$$p^- = - ik_t \frac{u^2 m c_s}{\rho} E_{11} \exp(ik_t x) .$$

$$V_t^+ = \frac{- i u c_s}{\rho_0 c} E_{01} \exp(- ik_t x) .$$

$$V_t^- = \frac{i u c_s}{\rho_0 c} E_{11} \exp(ik_t x) .$$

In these expressions,  $V_t^+$  and  $V_t^-$  are components in the positive x direction. Clearly,

$$\hat{z} = \frac{V_t^+}{p^+} = - \frac{V_t^-}{p^-} .$$

The expressions for  $V_L$  and  $p$  have been related to the corresponding electric fields.  $E_{Lr}$  is the reflected longitudinal electric field.

In order to establish meaningful power relations, it is convenient to examine the Poynting vector associated with these waves. If attention is restricted to the longitudinal waves, we have

$$P = \frac{1}{2} \operatorname{Re} [p V_L^*] = \frac{1}{2} \operatorname{Re} [p^+ (V_L^+)^* + p^- (V_L^-)^* + p^+ (V_L^-)^* + p^- (V_L^+)^*] \quad (23)$$

where  $\operatorname{Re}$  indicates the real part and the asterisk denotes complex conjugate. By introducing the reflection coefficient,  $\rho$ ,

$$\rho = \left. \frac{p^-}{p^+} \right|_{x=0} = \frac{E_{Lr}}{E_{Li}} \quad ,$$

(23) becomes

$$P = \frac{1}{2} \left\{ |p^+|^2 [\hat{R}(1 - |\rho|^2) + 2\hat{X} |\rho| \sin \psi] \right\} \quad (24)$$

at  $x = 0$ , the plane of density discontinuity. The terms introduced in (24) are defined as follows:

$$\hat{Z} = \hat{R} + i \hat{X} \quad , \quad (25)$$

$$\rho = |\rho| \exp(i\psi) \quad . \quad (26)$$

The first term in (24) clearly represents the difference between the powers carried by the two oppositely directed waves acting independently. The second term represents an interaction between the storage fields of the oppositely directed waves. The latter term is zero if either the impedance,  $\hat{Z}$ , or reflection coefficient,  $\rho$ , is real.

Equation (24) shows the inadequacy of calculating power by neglecting the interaction between incident and reflected fields. In some instances the error introduced is negligible, but in general is nonzero.

#### 4. Power Coupling at a Plasma-Vacuum Boundary

An analysis analogous to that carried out above may be made for a longitudinal wave incident on a boundary at oblique incidence. In that case a directional impedance may be defined which is dependent on the angle of incidence and the characteristics of the medium. If consideration is then given to the flow of power across a boundary, an expression similar to (24) can be obtained.

In establishing the equations for the latter study, a magnetostatic field may be included and set equal to zero if conditions dictate.

The equation satisfied by  $\bar{E}$  is [Callawa, 1965]

$$\begin{aligned} \nabla \times \nabla \times \bar{E} - k^2 \bar{E} - \frac{u^2}{v^2} \nabla \nabla \cdot \bar{E} + \frac{u^2}{v^2} \bar{E} \times \hat{z} \\ + \frac{u^2}{(v^2 + i\omega)} (\nabla \times \nabla \times \bar{E}) \times \hat{z} = 0 \end{aligned} \quad (27)$$



The angular electron gyrofrequency,  $\omega_c = -eB_0/m$ , has been introduced, where  $\vec{B} = \hat{k} B_0$  is the magnetic induction vector and  $\hat{k}$  is a unit vector.

The dispersion relation can be obtained by assuming a plane-wave solution which is proportional to  $\exp(-ik\hat{h} \cdot \vec{r})$ , where  $\hat{h}$  is a unit vector in the direction of propagation and  $\vec{r}$  is the vector to the variable point of interest. Under these conditions,

$$\nabla = -ik\hat{h}$$

and (25) reduces to

$$\begin{aligned} \hat{n} (\hat{n} \cdot \vec{E}) \left[ -k^2 + k^2 \frac{u^2}{v^2} \right] + \vec{E} \left[ k^2 - n^2 \right] \\ + \vec{E} \times \hat{a} \left[ \frac{i\omega\epsilon_0}{v} + \frac{k^2\epsilon_0}{v+i\omega} \right] - (\hat{n} \cdot \vec{E}) (\hat{n} \times \hat{a}) \frac{k^2\epsilon_0}{v+i\omega} = 0. \end{aligned} \quad (28)$$

This equation is quite complex when considered in its most general form. A tractable form can be deduced by restricting attention to the case whereby  $\hat{a}$  is perpendicular to  $\hat{h}$  and is such that  $\hat{h} \times \hat{a} = \hat{i}$ . Then  $\hat{n}$ ,  $\hat{a}$ ,  $\hat{i}$  form a right-handed triad and the basis of a rectangular coordinate system. Then

$$\vec{E} = \hat{n} E_n + \hat{a} E_a + \hat{i} E_i$$

and (25) reduces to three simultaneous equations:

$$E_0 \left[ k^2 \frac{u^2}{v^2} - n^2 \right] - E_1 \left[ \frac{i\omega\mu_1}{v^2} + \frac{\omega_1^2 k^2}{v^2 + i\omega} \right] = 0, \quad (28a)$$

$$E_0 (k^2 - n^2) = 0, \quad (28b)$$

$$E_0 \left[ -\frac{i\omega\mu_1}{v^2} \right] - E_1 (k^2 - n^2) = 0. \quad (28c)$$

If the magnetostatic field vanishes the medium can support three waves, each propagating independently. In general, however,  $E_0$  and  $E_1$  are coupled and purely transverse or longitudinal waves cannot be supported.

The set of equations (28a) and (28c) leads to solutions for  $k$  by equating the determinant of the coefficients to zero. This leads to

$$k^2 = \frac{n^2 + n^2 \frac{u^2}{v^2} - \frac{i\omega\mu_1^2}{v^2(v+i\omega)}}{2 \frac{u^2}{v^2}} \pm \frac{\sqrt{\left( -n^2 - n^2 \frac{u^2}{v^2} + i \frac{\omega\mu_1^2}{v^2(v+i\omega)} \right)^2 - 4 \frac{u^2}{v^2} \left( n^4 - \frac{\omega^2 \mu_1^2}{v^4} \right)}}{2 \frac{u^2}{v^2}}, \quad (29)$$

where the upper sign is associated with the quasi-longitudinal wave ( $k_l$ ) and the lower sign with the quasi-transverse wave ( $k_t$ ).

The way in which the complex directional impedance contributes to power coupling at a density discontinuity can be studied by examining the mechanism of reflection and transmission at a boundary. Here the boundary is taken as a plane surface, separating vacuum and plasma half-spaces. Attention is restricted to the case of coupling from an

electroacoustic (longitudinal or quasi-longitudinal) wave. In order to do this, a longitudinal wave is assumed to be incident on the boundary from the plasma (see fig. 1). Thus it is supposed that the incident wave is

$$\begin{aligned} \bar{E}_{i,1} = E_{i,1} \left[ \bar{a}_z \cos \theta + \bar{a}_x \sin \theta \right. \\ \left. + R (-\bar{a}_z \sin \theta + \bar{a}_x \cos \theta) \right] \exp \left[ -ik_1 (x \cos \theta + z \sin \theta) \right], \quad (30) \end{aligned}$$

where

$$R = \frac{-i\omega\mu_0}{v^2 (k_1^2 - \kappa^2)}. \quad (31)$$

The reflected quasi-longitudinal and quasi-transverse waves are, respectively,

$$\begin{aligned} \bar{E}_{r,1} = E_{r,1} \left[ -\bar{a}_z \cos \theta + \bar{a}_x \sin \theta \right. \\ \left. + R (-\bar{a}_z \sin \theta - \bar{a}_x \cos \theta) \right] \exp \left[ -ik_1 (-x \cos \theta + z \sin \theta) \right], \quad (32) \end{aligned}$$

$$\begin{aligned} \bar{E}_{r,2} = E_{r,2} \left[ -\bar{a}_z \sin \vartheta_1 - \bar{a}_x \cos \vartheta_1 \right. \\ \left. + T (-\bar{a}_z \cos \vartheta_1 + \bar{a}_x \sin \vartheta_1) \right] \exp \left[ -ik_{1,1} (-x \cos \vartheta_1 + z \sin \vartheta_1) \right], \quad (33) \end{aligned}$$

where

$$T = \frac{(k_{1,1}^2 - \kappa^2) v^2}{-i\omega\mu_0}. \quad (34)$$

The transmitted wave is

$$\bar{E}_{t,1} = E_{t,1} [\bar{A}_1 \sin \theta_2 - \bar{A}_2 \cos \theta_2] \exp [-ik_{t,2}(x \cos \theta_2 + z \sin \theta_2)] \quad (35)$$

where  $\theta$  and  $\varphi$  are used in association with longitudinal and transverse waves, respectively. The subscript 1 or 2 is used to designate plasma or vacuum, respectively.

The boundary conditions used to determine the amplitudes are continuous (zero) normal electron velocity and continuous tangential electric and magnetic fields.

Using (8) and (15), expressions for  $p$  and  $\bar{V}_t$  can be obtained in terms of the electric field. Expressions for the directional impedance may then be defined in terms of these expressions. To do so, superscript + or - will be attached to designate incident and reflected waves, respectively. Thus, with the magnetostatic field taken to be zero,

$$p^+ = \frac{-ik_1 u^2 m c_0}{e} E_{t,1} \exp [-ik_1(x \cos \theta + z \sin \theta)] \quad (36)$$

$$p^- = \frac{-ik_1 u^2 m c_0}{e} E_{t,1} \exp [-ik_1(-x \cos \theta + z \sin \theta)] \quad (37)$$

$$\bar{V}_t^+ = \frac{-izc_0}{n_0 e} E_{t,1} [\bar{A}_1 \cos \theta + \bar{A}_2 \sin \theta] \exp [-ik_1(x \cos \theta + z \sin \theta)] \quad (38)$$

$$\bar{V}_t^- = \frac{-izc_0}{n_0 e} E_{t,1} [-\bar{A}_1 \cos \theta + \bar{A}_2 \sin \theta] \exp [-ik_1(-x \cos \theta + z \sin \theta)] \quad (39)$$

The reflection coefficient may be defined as before:

$$\rho = \left. \frac{p^-}{p^+} \right]_{z=0} = \frac{E_{1r}}{E_{1i}} \quad (40)$$

Directional electroacoustic wave impedance may then be defined in a method analogous to that previously discussed. Thus, we take

$$\hat{Z} = \frac{V_{11}^+}{p^+} = - \frac{V_{11}^-}{p^-} \quad (41)$$

where

$$\bar{V}_1^+ = \bar{a}_1 V_{11}^+ + \bar{a}_2 V_{21}^+ \quad .$$

$$\bar{V}_1^- = \bar{a}_1 V_{11}^- + \bar{a}_2 V_{21}^- \quad .$$

Then the directional impedance,  $\hat{Z}_1$ , is given by

$$\hat{Z}_1 = \frac{\hat{Z} \cos \theta}{\sqrt{1 - \frac{u_z^2}{c^2} - 1 v/u}} = \hat{Z} \cos \theta \quad (42)$$

$T_z$  - time-average x-component of the Poynting vector associated with the electroacoustic wave then becomes

$$P_z = \frac{1}{2} |p^+|^2 \left\{ \hat{R}_1 (1 - |\rho|^2) + 2 \hat{X}_1 e^{+i\phi_x} (\alpha \sin \gamma x + \beta \cos \gamma x) \right\} \quad (43)$$

where

$$\hat{Z}_1 = \hat{R}_1 + i \hat{X}_1 \quad .$$

$$\rho = \alpha + i\beta \quad .$$

$$2 k_1 \cos \theta = \gamma - i \xi_1 \quad .$$

Equation (43), which is valid for  $x \leq 0$ , illustrates that the Poynting vector has an exponentially decaying standing wave component. This component of the vector is established by the reactive component of the directional impedance. At  $x = 0$ , (43) becomes

$$P_1 = \frac{1}{2} |p^+|^2 \{ \hat{R}_1 (1 - |\rho|^2) + 2 \hat{X}_1 \beta \} \quad . \quad (44)$$

which is entirely analogous to (24) with directional impedance replacing impedance.

Attention has been, up to now, restricted to the power flow due to the electroacoustic wave. A contribution due to the electromagnetic waves will also be present. If a magnetostatic field permeates the region, an electromagnetic wave necessarily accompanies the electroacoustic wave. In any case a reflected and a transmitted electromagnetic wave are generated at the boundary, as in (30)-(35).

The power flow described in (24) and (44) is quite analogous to that described by Earlow [1963] for the transverse electromagnetic waves.

In order to describe the power coupling mechanism at the boundary, we choose here to examine the magnitudes of the Poynting vectors in the vacuum and that associated with the incident wave. The ratio of the former to the latter will be termed the power conversion factor (see figs. 6 - 13). With this definition, the power conversion factor is not physically meaningful except that it illustrates phenomena specifically related to the complex electroacoustic wave impedance. In particular, it is evident from figure 6 that the interaction of the incident and reflected waves is significant at the lower frequencies, where the last term in (44) predominates. For example, if  $\omega_p \gg \omega$ ,  $v \gg \omega$  are both satisfied and in addition  $(\omega_p^2 / \omega v) \gg 1$ , it is not difficult to show that (see (44))

$$\frac{2\hat{X}_1 \theta}{R_1} \approx 8 \frac{u}{c} \frac{\omega_p}{v} \frac{\sin^2 \varphi_0}{\cos \varphi_0}.$$

This exceeds unity when  $\varphi_0$  exceeds about  $80^\circ$  and  $(\omega_p / v) > 20$  ( $u/c$  is  $10^{-8}$  throughout).

The power conversion factor, as herein defined, is useful in illustrating the effects of a complex wave impedance; at the same time, it is somewhat superficial because it attempts to separate power contained in the incident wave from the total incident power when in fact the interaction

of the incident and reflected waves precludes this separation. There is, then, a distinct difference between the power in the incident wave and the total incident power. Indeed, the total incident electroacoustic power can be considerably greater than the power contained in the incident wave (see fig. 6). Thus, for the electroacoustic mode, the total power through a given cross-section may be obtained by integrating the power density, as given in (44). With the normalization chosen here, attention is restricted to an examination of the power density in the vacuum due to an incident longitudinal wave maintained at unity magnitude ( $E_0 = 1$ ).

In the plasma, the time average Poynting vector is [Chen, 1964; Field, 1956]

$$\bar{S}_w = \frac{1}{2} \text{Re}[\bar{E} \times \bar{H}^* + p\bar{V}^*] \quad (45)$$

where Re indicates the real part, and the asterisk specifies the complex conjugate. Using (2), (3), and (4), along with the assumed field variation, the Poynting vector for the incident quasi-longitudinal wave becomes



$$\begin{aligned}
\bar{S}_{x,v} = & \frac{|E_{x1}|^2}{2\omega\mu_0} |R|^2 [\bar{E}_1 \operatorname{Re}(k_1^* \cos \theta) - \bar{E}_1 \operatorname{Re}(k_1^* \sin \theta)] \\
& + \frac{|E_{x1}|^2}{2\omega\mu_0} [\bar{E}_1 \operatorname{Re}(R^* k_1^* \sin \theta) - \bar{E}_1 \operatorname{Re}(R^* k_1^* \cos \theta)] \\
& - \frac{u^2}{\omega^2} \frac{|E_{x1}|^2}{2} [\bar{E}_1 \operatorname{Re}(-k_1 \omega \epsilon_0 (\cos \theta)^*) + \bar{E}_1 \operatorname{Re}(-k_1 \omega \epsilon_0 (\sin \theta)^*) \\
& + \bar{E}_1 \operatorname{Re}\left(\frac{k_1 R^* (\sin \theta)^*}{\omega\mu_0} \{-(k_1^*)^2 + \omega^2 \mu_0 \epsilon_0\}\right) \\
& + \bar{E}_1 \operatorname{Re}\left(\frac{k_1 R^* (\cos \theta)^*}{\omega\mu_0} \{(k_1^*)^2 - \omega^2 \mu_0 \epsilon_0\}\right)] \quad (46)
\end{aligned}$$

In the vacuum

$$|\bar{S}_{x,v}| = \frac{1}{2} \operatorname{Re} \bar{E} \times \bar{H}^* = \frac{1}{2} \left| \frac{E'_{x1}}{E_{x1}} \right|^2 |E_{x1}|^2 \frac{k_{x2}}{\omega\mu_0} \quad (47)$$

In the absence of a magnetostatic field, (46) reduces to

$$\bar{S}_{x,v} = \frac{\omega \epsilon_0 u^2}{\omega^2} \frac{|E_{x1}|^2}{2} [\bar{E}_1 \operatorname{Re}(k_1 (\cos \theta)^*) + \bar{E}_1 \operatorname{Re}(k_1 (\sin \theta)^*)] \quad (48)$$

## 5. Discussion

The electroacoustic wave impedance is generally complex unless collisions are neglected. The associated x-directed component of the Poynting vector at the boundary is therefore made up of two parts. The first part consists of the difference in the magnitudes of the forward and

backward traveling waves. The second part is due to the interaction of the reactive components. Conceivably, the latter portion of the Poynting component can exceed the former, if the imaginary parts of the various terms are of sufficient magnitude. This is generally the case for  $\omega < \omega_p$ .

The imaginary part of the wave impedance exceeds or is less than the real part depending on whether  $\omega < \omega_p$  or  $\omega > \omega_p$ . At  $\omega = \omega_p$ , the two components are equal. The manner in which the imaginary part changes with frequency for  $\omega > \omega_p$  depends on the electron density and the collision frequency. For the higher collision frequencies, the imaginary and real parts of the impedance tend to remain almost equal for  $\omega > \omega_p$ . For lower collision frequencies, this tendency does not prevail, and the imaginary part drops off quite rapidly with frequency for frequencies greater than the plasma frequency. These factors are illustrated graphically in figures 2 - 5. The value of  $\frac{u}{c}$ , where  $c$  is the velocity of light, is taken to be  $10^{-3}$  throughout. This corresponds to a temperature of about  $10^6$  degrees Kelvin.

The variation of the power conversion factor, as earlier defined, is given in figures 6 - 13. For  $v = 10^8$  the power conversion factor exceeds unity for frequencies somewhat less than the plasma frequency. This is not surprising in view of the earlier discussion. The curves are parametric in electron density and are given as functions of frequency and (real) angle  $\phi_0$ . The results are given for the case of zero

magnetostatic field (figs. 6 - 9) and also (figs. 10 - 13) for the case of a magnetostatic field having a magnitude of  $5 \times 10^{-8}$  Webers/m<sup>2</sup> (0.5 Gauss). In the latter case there is necessarily a nonzero (transverse) electromagnetic field associated with the incident wave. There will therefore be a wave interaction of the type described by Barlow [1963] as well. This interaction will be due to the complex wave impedance seen by that wave. The graphs showing the E-field conversion factor corresponding to figures 6 - 13 have been given in an earlier report [Callawa, 1965].

## 6. Conclusions

The results presented in this paper indicate that the reactive component of the wave impedance, as seen by an electroacoustic wave, may play an important role in the power coupling at a plasma-vacuum boundary. The results show that the storage fields of oppositely directed waves may interact to produce a significant flow of power. Thus, the total flow of power may differ appreciably from that due to the incident and reflected waves acting independently. This is generally the case for frequencies less than the plasma frequency.

An important question which remains to be answered pertains to the effectiveness of the boundary as a power coupling device for the two modes of incident waves; i. e., one should examine the effectiveness for incident electromagnetic and electroacoustic waves. Because of

the importance of the complex wave impedance, it is probable that one mode provides more power flow across the boundary than the other for the same incident power. This possibility has its corollary in the problem of evanescent fields in hollow metallic guides. In that case Barlow [1963] found that the E mode can provide a much larger flow of power than the H mode for the same applied field and terminal impedance. The question as it applied to plane waves in a plasma will be examined in a future report.

## 7. Glossary

$h$	$u_y^2 / (u^2 + v^2)$
$m$	mass of an electron
$n_e$	electron number density
$t$	time
$\vec{v}$	velocity vector
$e$	electronic charge
$\vec{E}$	electric field vector
$E$	electric field magnitude
$\vec{B}$	static magnetic flux density vector
$\vec{H}$	magnetic field vector
$p$	thermodynamic perturbation pressure
$u$	speed of sound in electron gas
$i$	$\sqrt{-1}$
$v$	a term defined in equation (14)
$R$	a term defined in equation (31)
$\hat{R}$	real part of electroacoustic wave impedance
$\hat{X}$	imaginary part of electroacoustic wave impedance
$\hat{Z}$	$\hat{R} + i \hat{X}$
$\hat{Z}_0$	acoustic wave impedance
$T$	a term defined in equation (34)
$\hat{n}, \hat{x}, \hat{m}, \hat{t}$	unit vectors

$\mathbf{r}$	vector to a variable point
$k$	wave number
$c$	velocity of light
$\exp$	exponential function
$x, y, z$	the Cartesian coordinate variables
$\nu$	collision frequency
$\mu_0$	permeability of space
$\epsilon_0$	permittivity of space
$\omega$	radian frequency
$\omega_p$	plasma frequency
$\omega_c$	electron cyclotron frequency
$\alpha$	a term defined in equation (12)
$\eta$	index of refraction
$\theta$	direction angles associated with electroacoustic waves
$\phi$	direction angles associated with (transverse) electromagnetic waves
$\rho_v$	volume mass density
$\rho$	reflection coefficient
$\psi$	phase angle of $\rho$

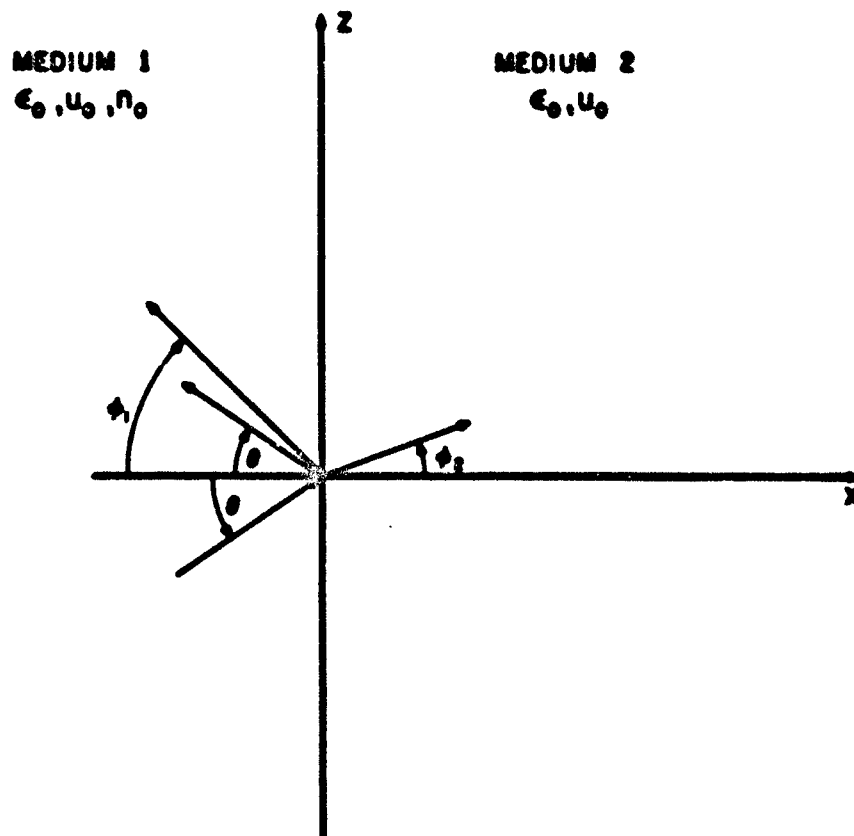
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**Figure 1.** Coordinate system showing the two media and illustrating the angles of incidence, reflection, and transmission.

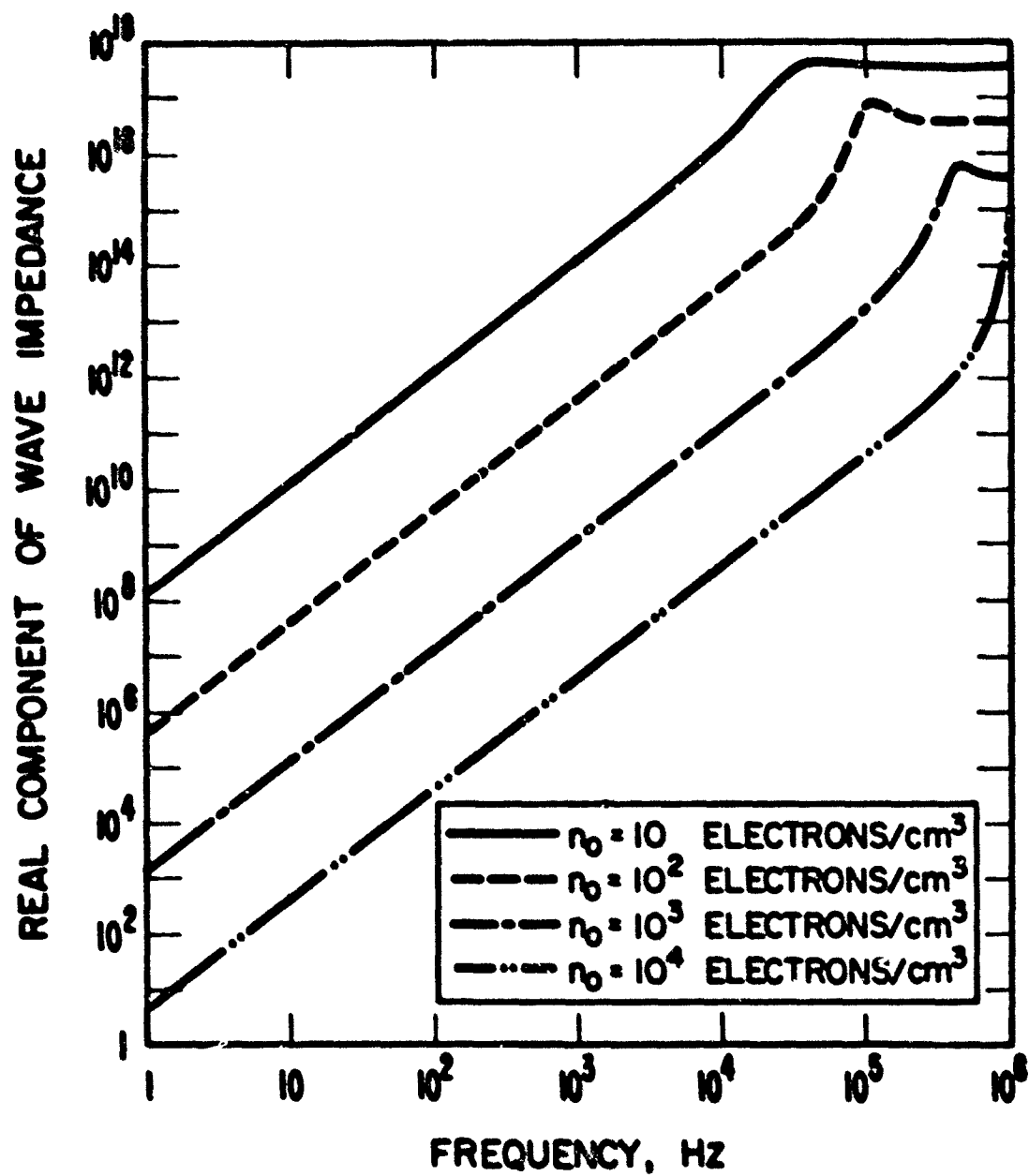


Figure 2. Real component of electroacoustic wave impedance.  
 $\nu = 10^9 \text{ sec}^{-1}$

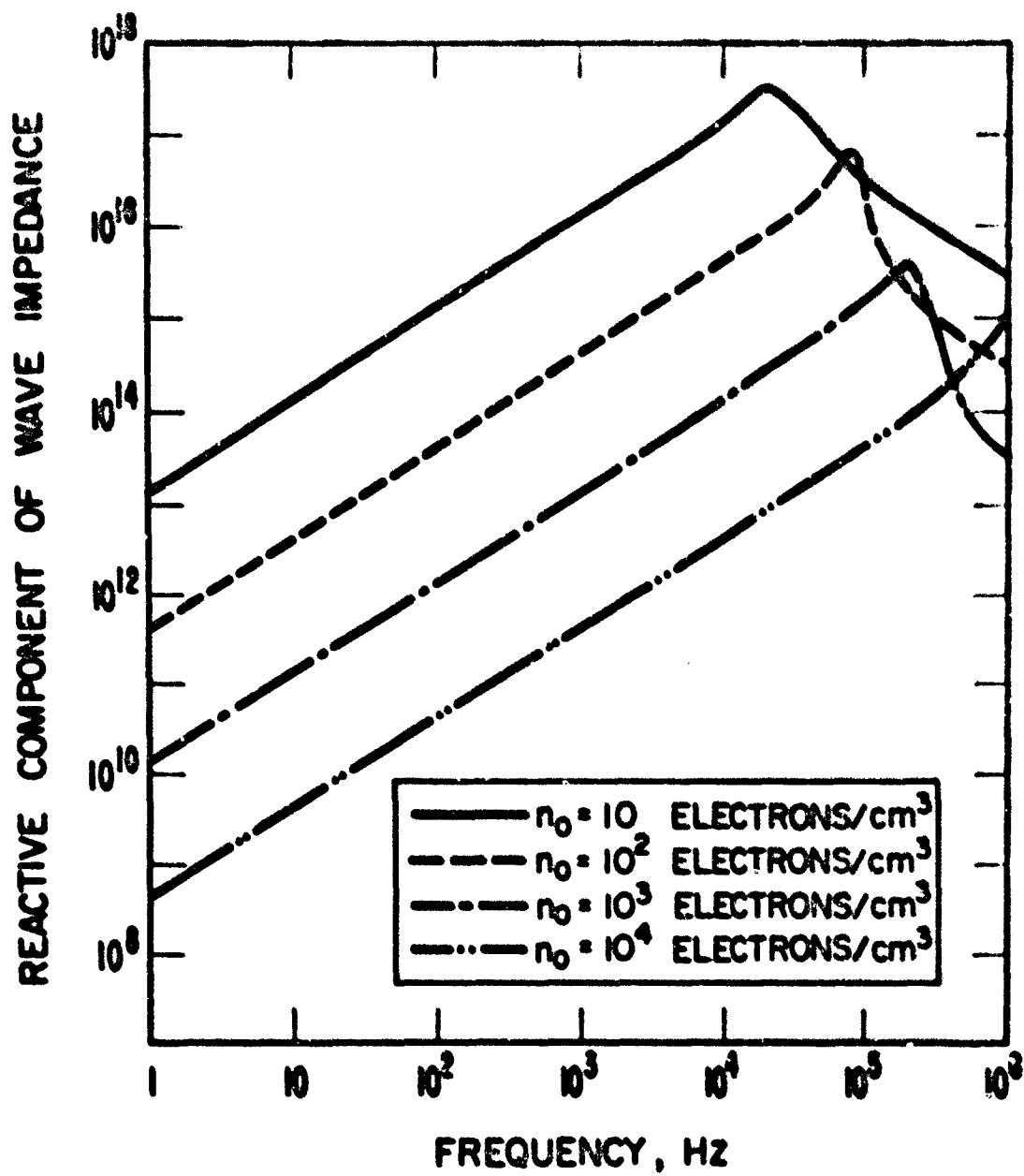


Figure 3. Imaginary component of electroacoustic wave impedance.  $v = 10^8 \text{ sec}^{-1}$

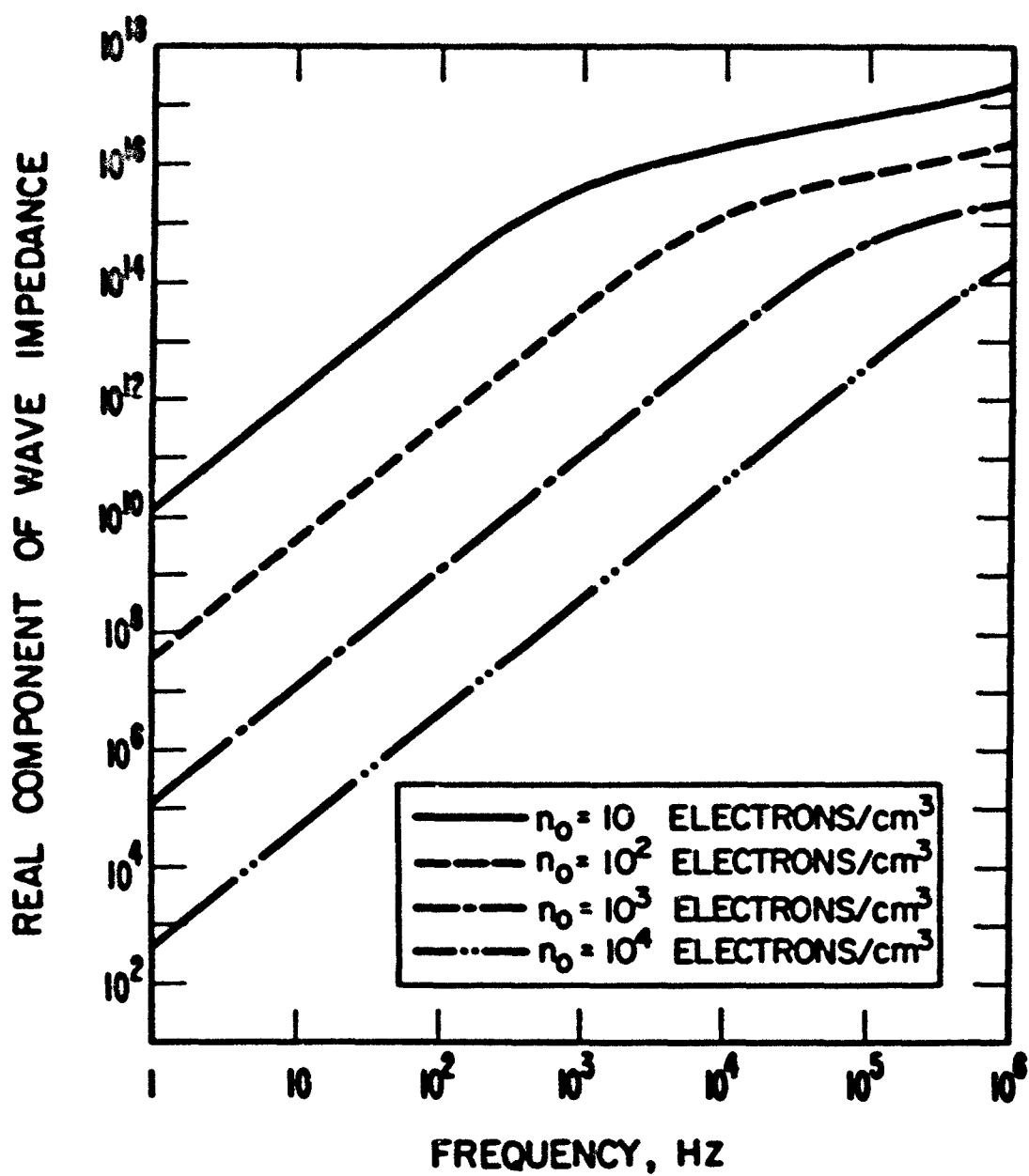


Figure 4. Real component of electroacoustic wave impedance.  
 $\nu = 10^7 \text{ sec}^{-1}$

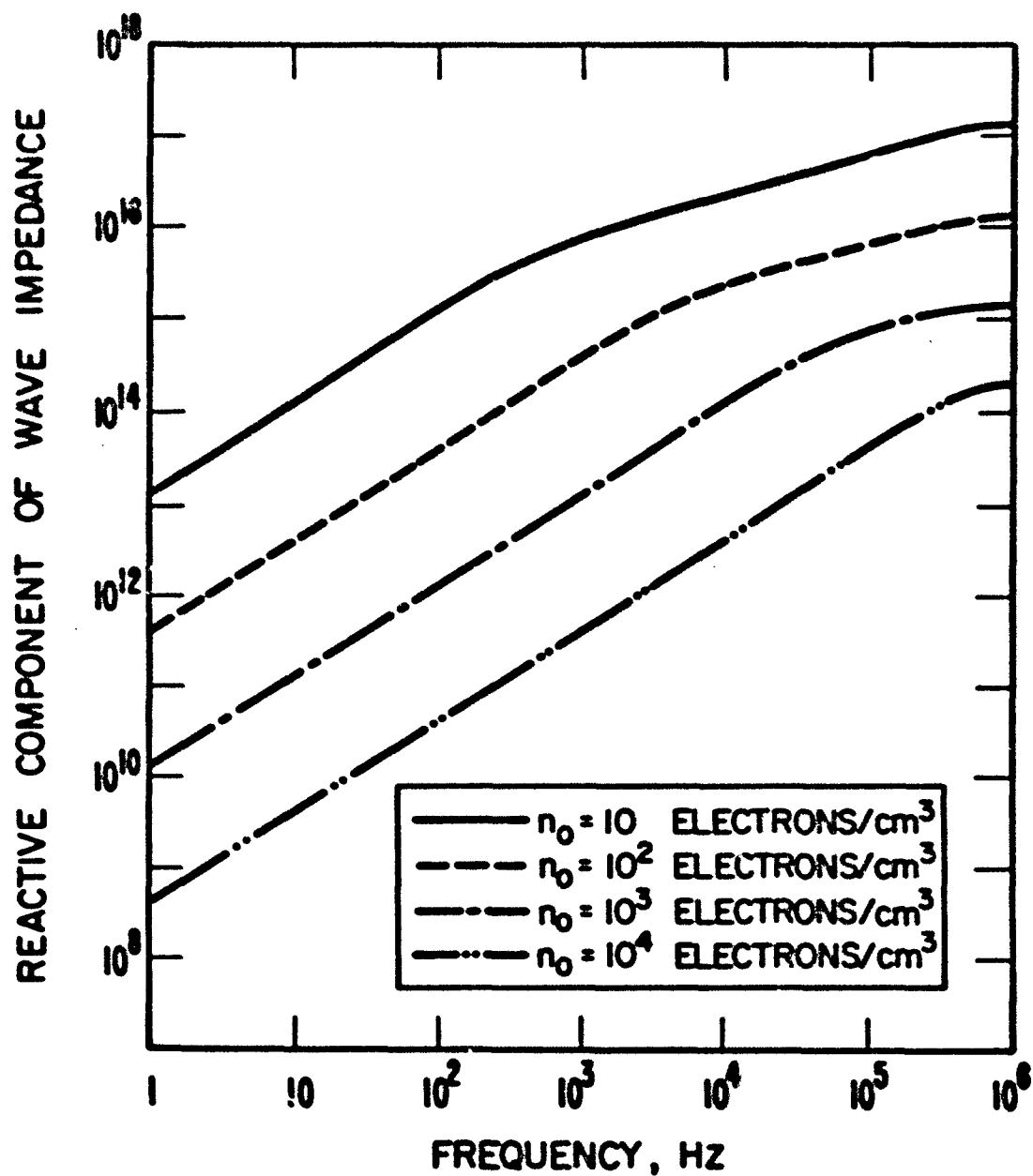


Figure 5. Imaginary component of electroacoustic wave impedance.  $\nu = 10^7 \text{ sec}^{-1}$ .

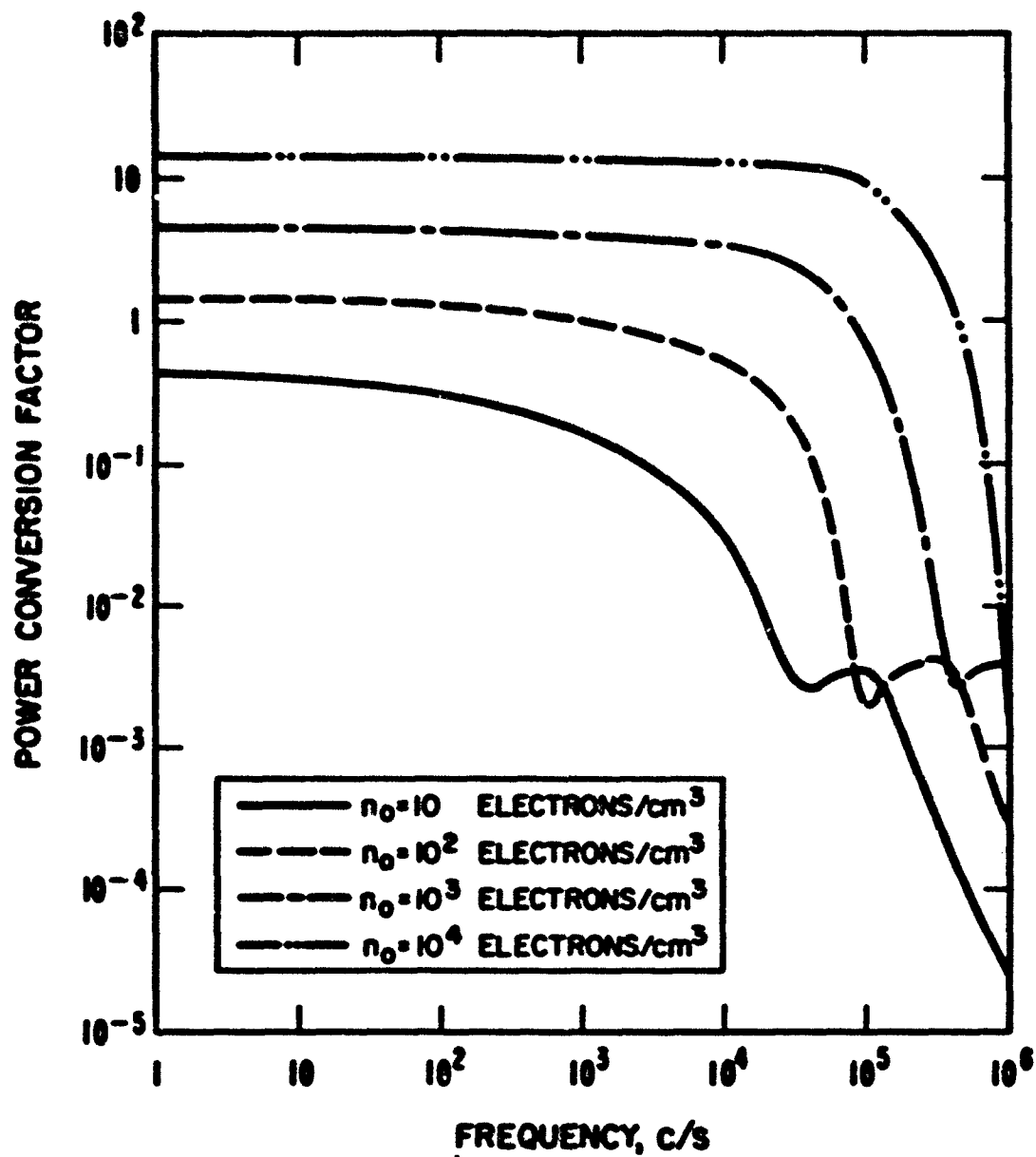


Figure 6. Dimensionless power conversion factor.  $B_0 = 0$ ;  
 $\phi_2 = 80^\circ$ ;  $\nu = 10^8 \text{ sec}^{-1}$

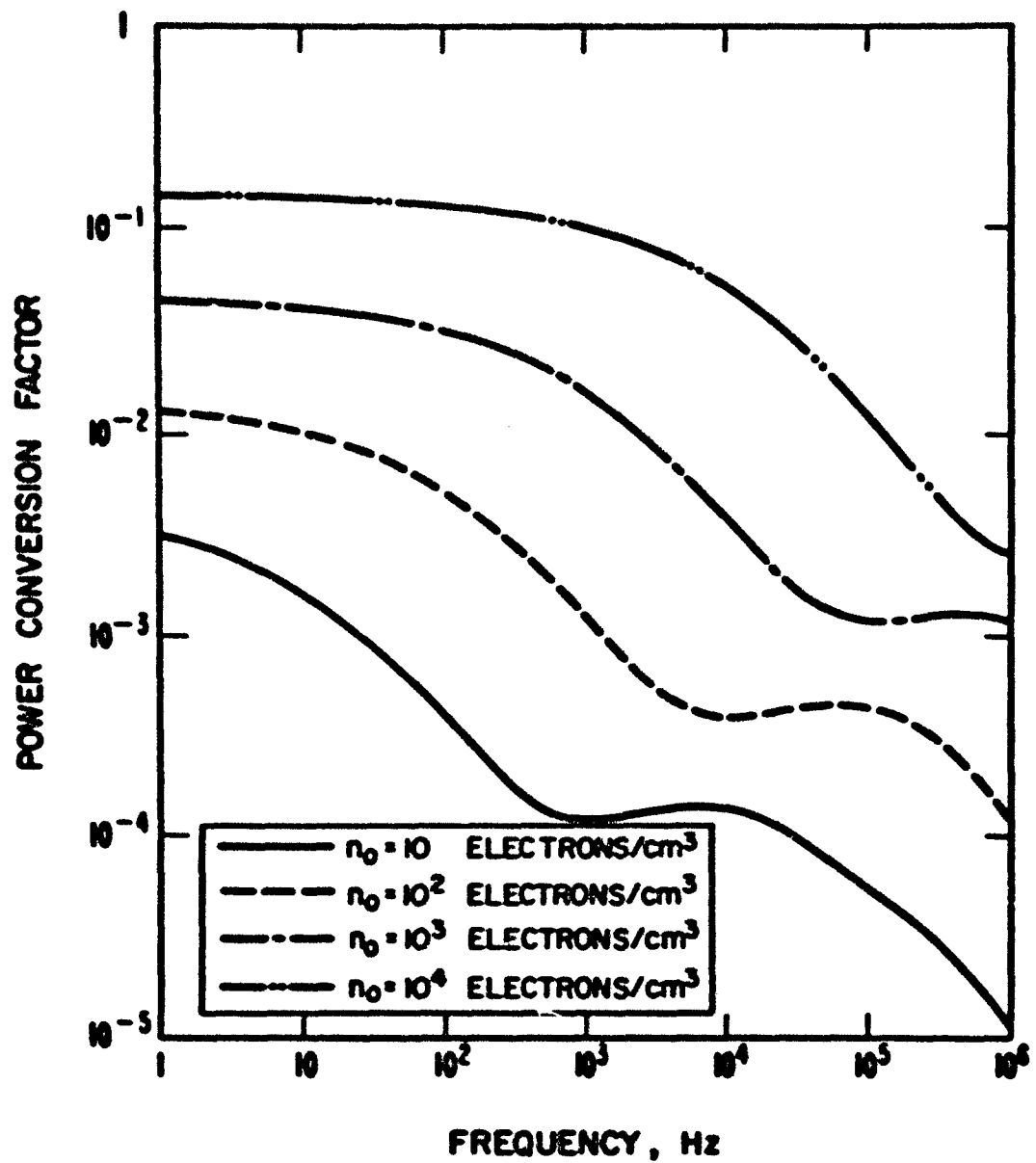


Figure 7. Dimensionless power conversion factor.  $B_0 = 0$ ;  
 $\theta_0 = 80^\circ$ ;  $\nu = 10^7 \text{ sec}^{-1}$ .

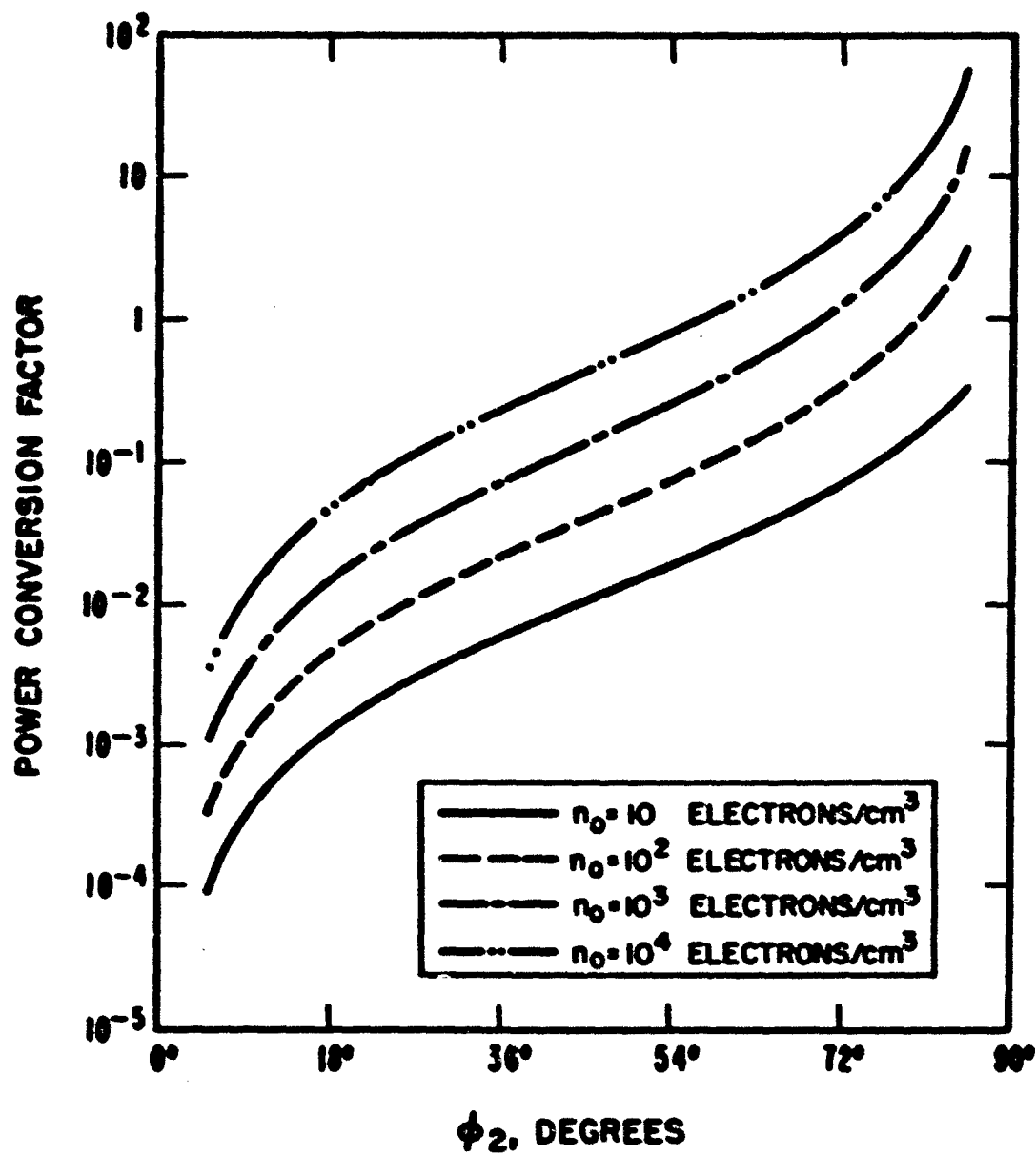


Figure 8. Dimensionless power conversion factor.  $B_0 = 0$ ;  
 $f = 1$  kHz;  $\nu = 10^5$  sec<sup>-1</sup>



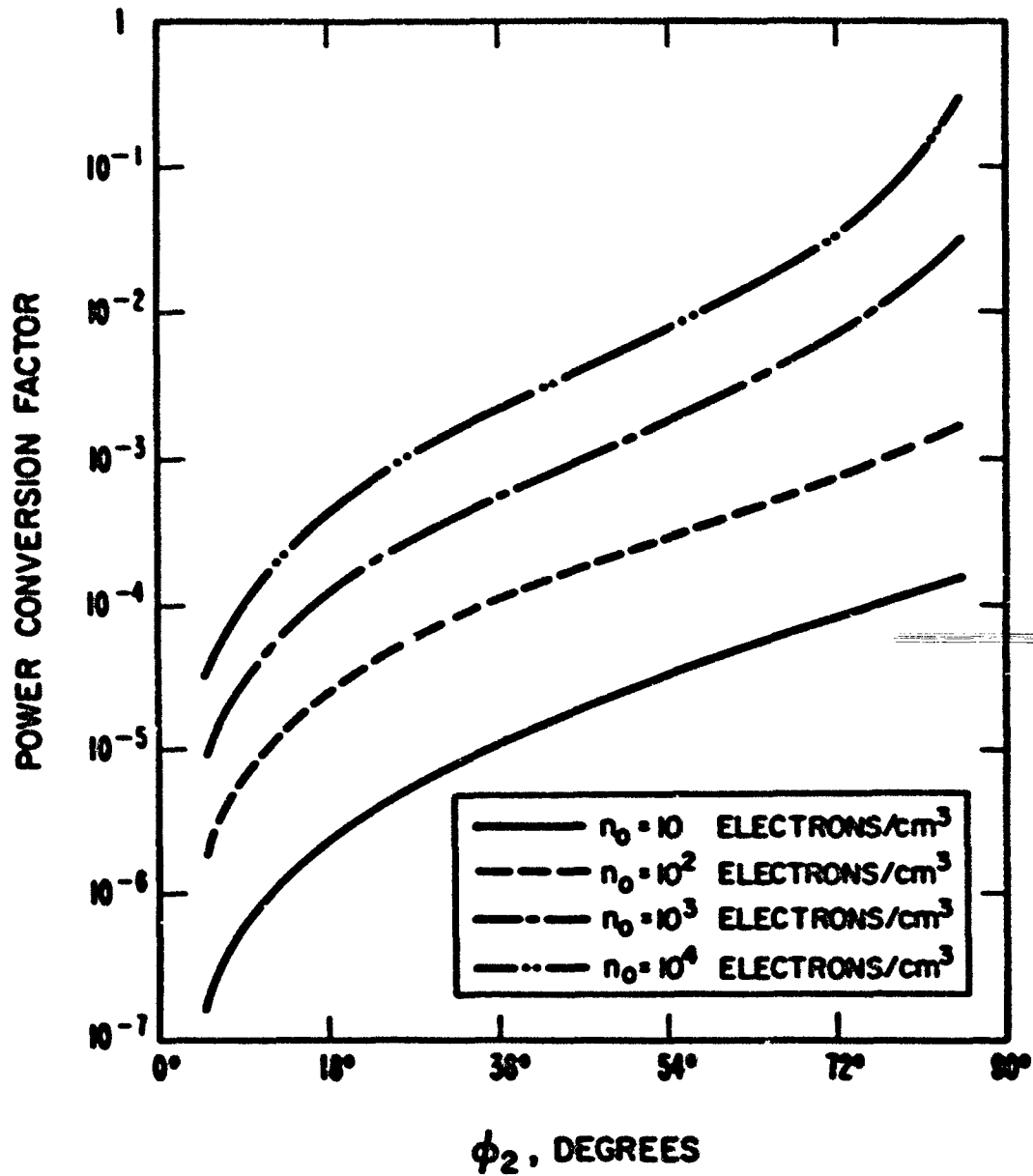


Figure 9. Dimensionless power conversion factor.  $B_0 = 0$ ;  
 $f = 1$  kHz;  $\nu = 10^7$  sec<sup>-1</sup>.

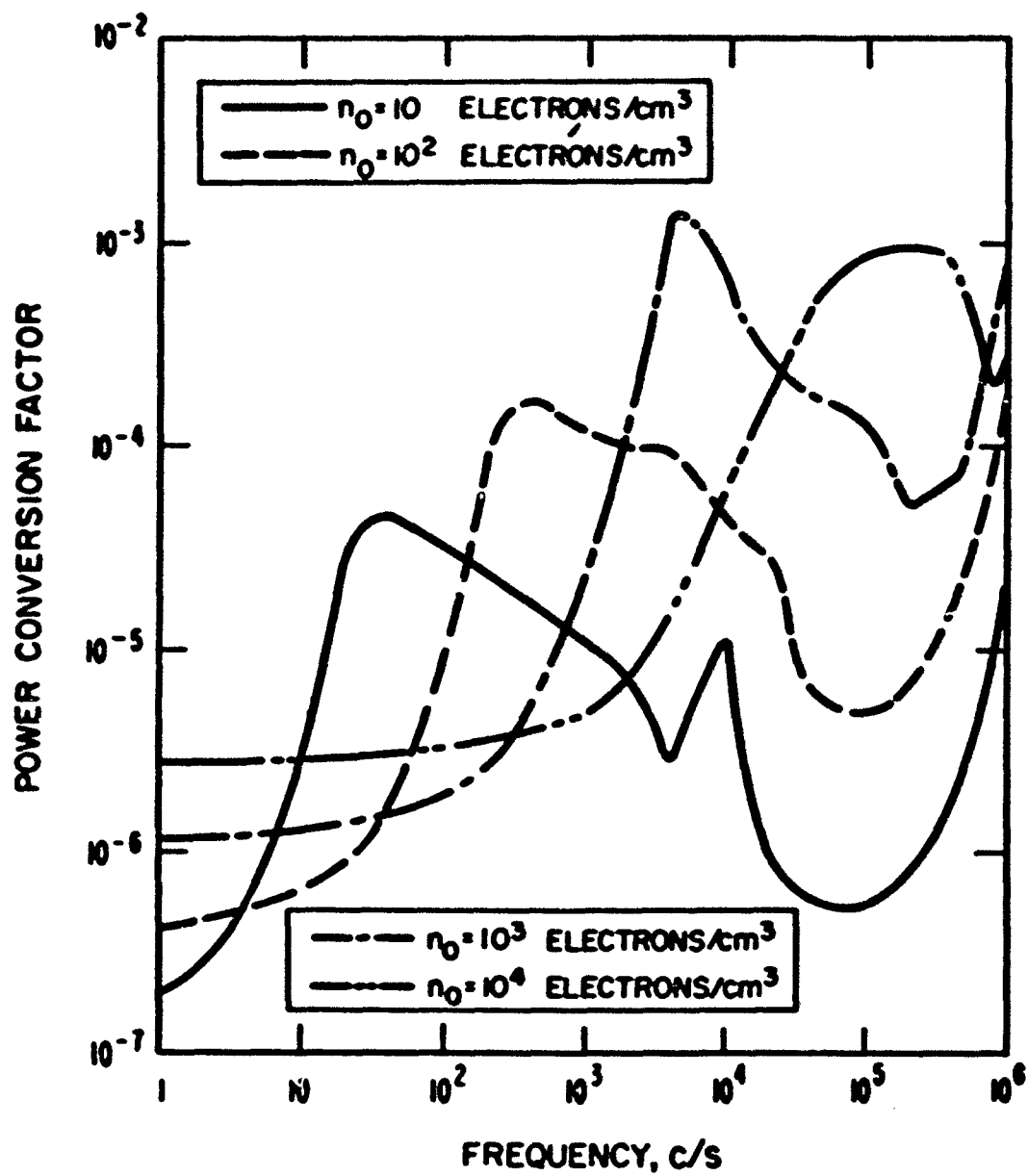


Figure 10. Dimensionless power conversion factor.  $B_0 = 5 \times 10^{-8}$  Wb/m<sup>2</sup>;  $\phi_2 = 80^\circ$ ;  $\nu = 10^8$  sec<sup>-1</sup>.

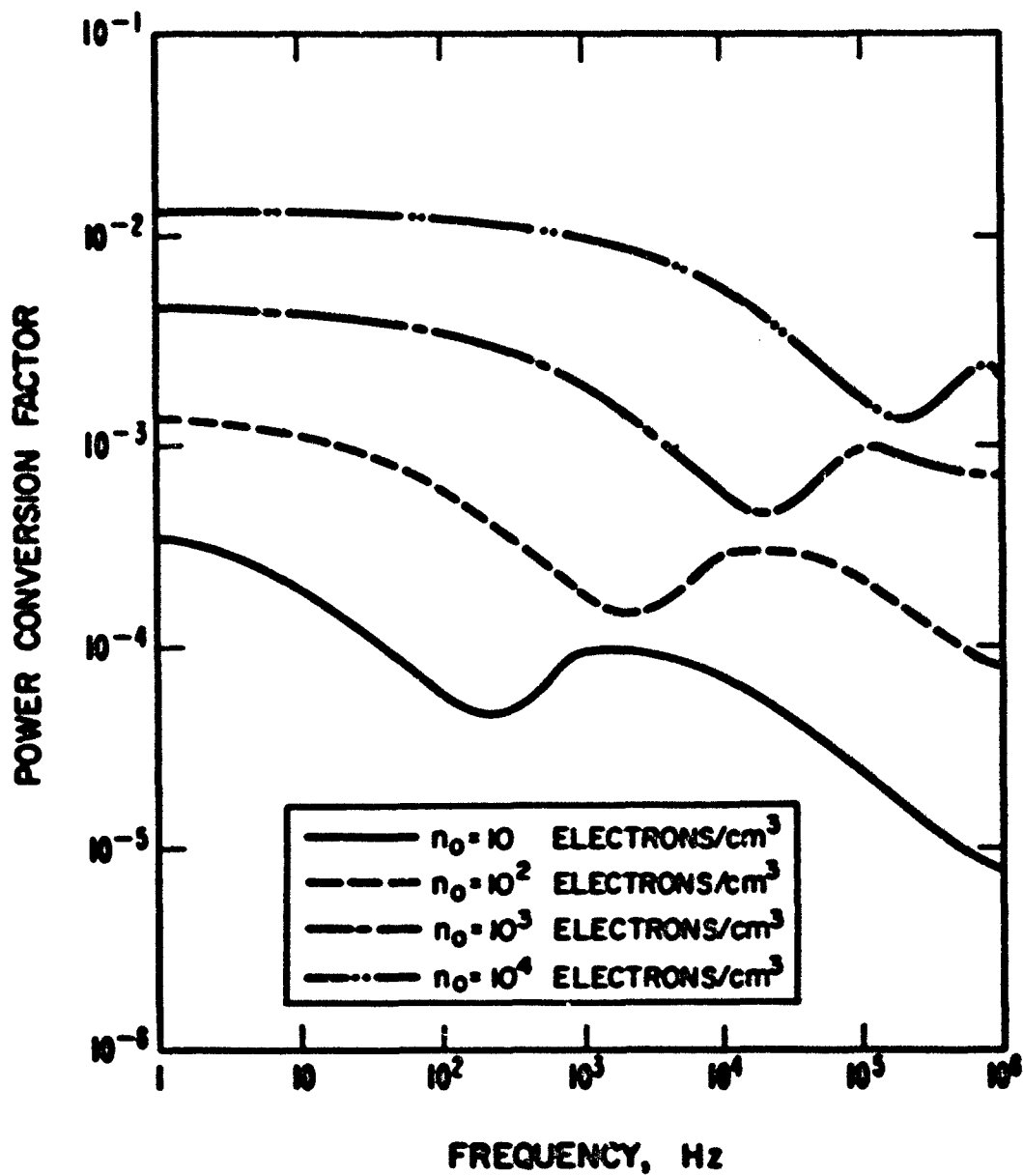


Figure 11. Dimensionless power conversion factor.  $B_0 = 5 \times 10^{-8}$  Wb/m<sup>2</sup>;  $\omega_0 = 80^\circ$ ;  $\nu = 10^7$  sec<sup>-1</sup>.

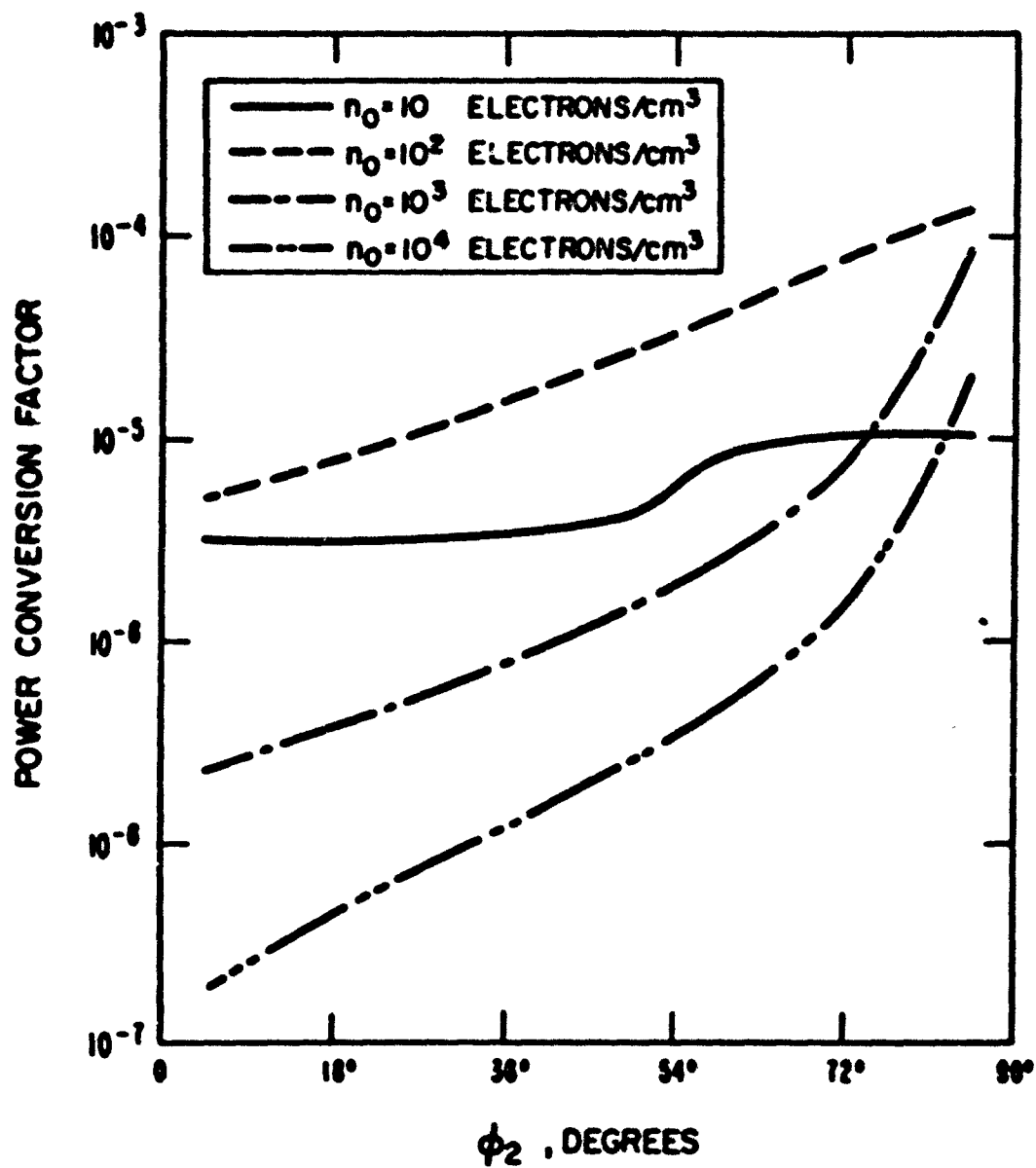


Figure 12. Dimensionless power conversion factor.  $B_0 = 5 \times 10^{-6}$  Wb/m<sup>2</sup>;  $f = 1$  kHz;  $\nu = 10^8$  sec<sup>-1</sup>.

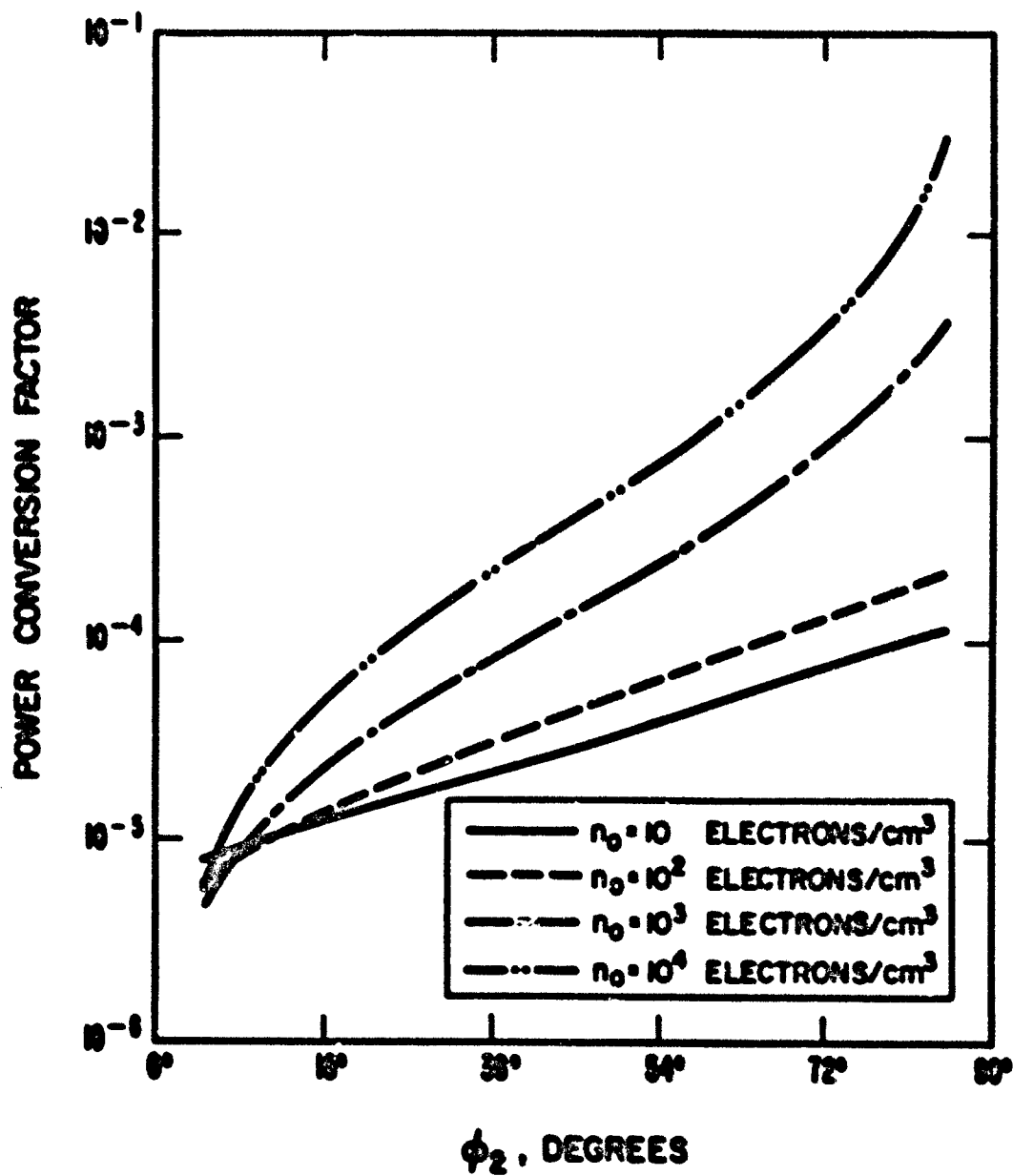


Figure 13. Dimensionless power conversion factor.  $B_0 = 5 \times 10^{-6}$  Wb/m<sup>2</sup>;  $f = 1$  kHz;  $\nu = 10^7$  sec<sup>-1</sup>.

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